Please paste the barcoded label here	TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2021

MATHEMATICS: PAPER I

EXAMINATION NUMBER													

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Write your answers in the spaces provided.
5. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
6. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers.

   Answers only will NOT necessarily be awarded full marks.
7. Diagrams are not necessarily drawn to scale.
8. If necessary, round off answers to **ONE** decimal place, unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. Three blank pages (pages 22 to 24) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	TOTAL
20	14	12	12	9	9	9	15	11	18	8	13	/150

SECTION A

QUESTION 1

(a) (1) Use the formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to solve for x correct to two decimal places if $3x^2 - 5x = 3$.

(4)

(2) Hence, solve for x if $3x^2 - 5x - 3 > 0$.

(2)

(b) Solve for x and y if $2y - 24x = 0$ and $y = x^2 + 5x$.

(5)

(c) Solve for x if $6+\sqrt{x+7}=x+1$.

(5)

(d) If the first three terms of a geometric series are 2 + 6 + 18 + … then what is the value of n if $S_n = 177\,146$?

(4)

[20]

QUESTION 2

(a) Using first principles find $f'(x)$ if $f(x) = x^2 - 5x$.

(5)

(b) Find $g'(x)$ if $g(x) = \sqrt[3]{x} + \frac{6}{x}$.

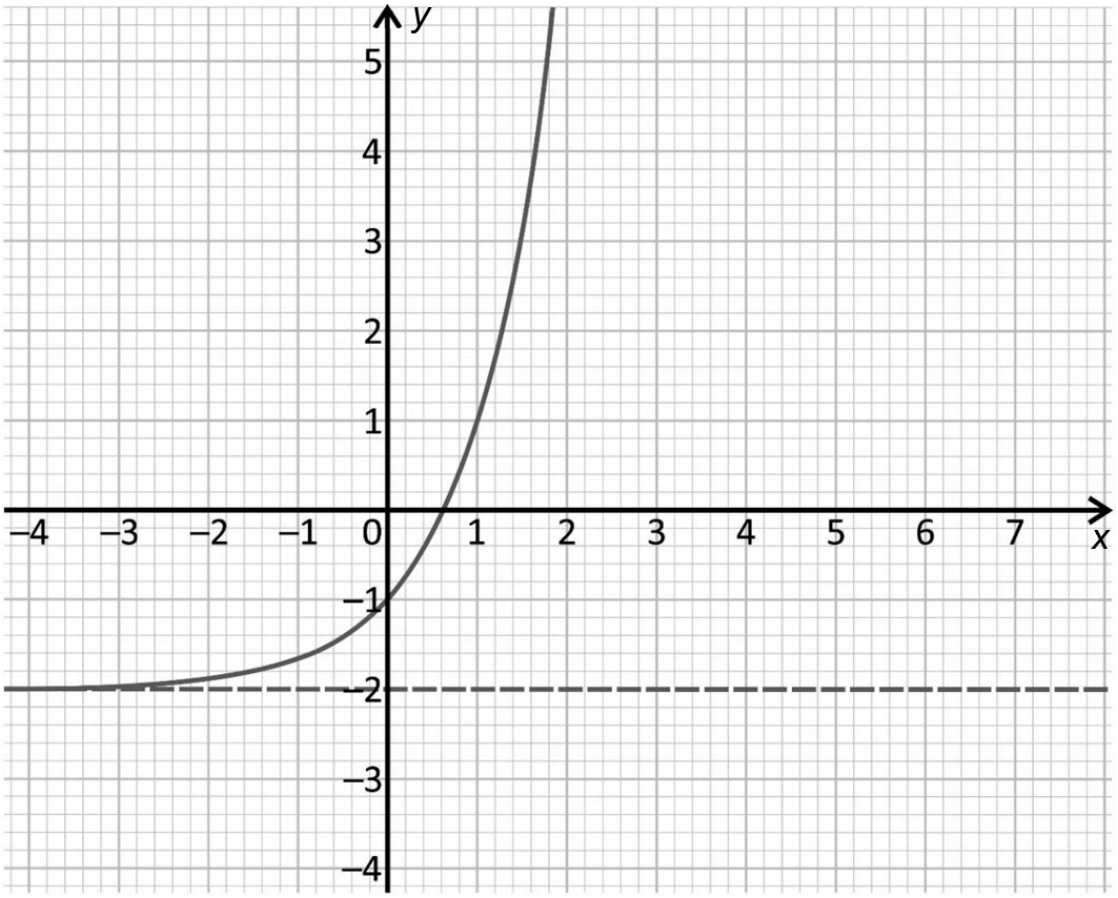
(4)

(c) If $h(x) = -x + p$ is a tangent to $f(x) = -x^2 + 3x + 4$ then find the value of p.

(5)
[14]

QUESTION 3

The graph of $f(x) = 3^x - 2$ has been drawn on the set of axes below.



(a) (1) Write down the domain of *f*.

(1)

(2) Write down the range of *f*.

(1)

(3) Write down the equation of the graph that is a reflection of *f* in the *x*-axis.

(1)

(4) Determine the values of x for which $f(x) \geq 0$.

(3)

(b) (1) On the same set of axes above sketch $g(x)$, if $g(x) = -2^x + 4$.

(3)

(2) Determine the equation of the resulting graph if g is shifted 3 units vertically up.

(1)

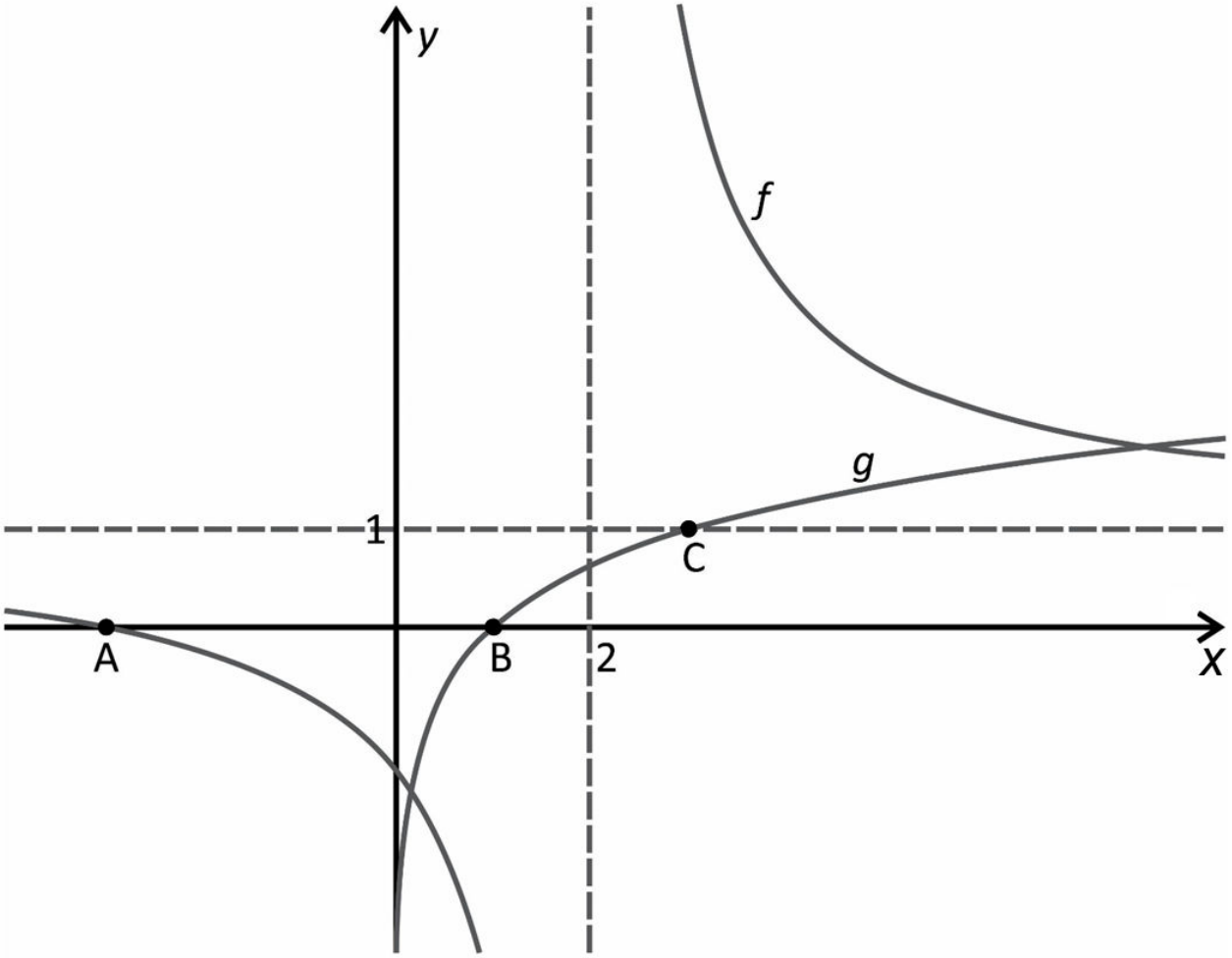
(3) Show on your graph where you would read off the solution to $3^x + 2^x = 6$.

(2)
[12]

QUESTION 4

In the diagram below:

- $f(x) = \frac{5}{x-p} + q$ and $g(x) = \log_3(x)$ are drawn.
- $x = 2$ is an asymptote for $f(x)$.
- $y = 1$ passes through C and is an asymptote for $f(x)$.



(a) If $h(x) = x + t$ is an axis of symmetry for $f(x)$, then determine the value of t.

(2)

(b) Calculate the x-coordinate of C.

(1)

(c) Find the length of AB.

(5)

(d) Write the equation for $g^{-1}(x)$ in the form $y = ...$

(2)

(e) For what values of x is $f(x).g(x) < 0$?

(2)

[12]

QUESTION 5

Refer to the pattern below:

$4+9+18+31+48+\ldots+949$

(a) Determine T_n.

(4)

(b) Determine the number of terms in the series.

(3)

(c) Represent the series $4+9+18+31+48+\ldots+949$ using ***sigma*** notation.

(2)
[9]

QUESTION 6

The details of an investment opportunity with a company called Phantom Investment House are given below:

- Monthly payments of R5 000 need to be made at the end of each month.
- The interest rate offered is 15% per annum compounded monthly.

You decide to invest with Phantom Investment House with the terms given above.

(a) What will the value of your investment be at the end of 3 years?

(4)

(b) What are the minimum number of months required for your investment to reach a value greater than R2 500 000?

(5)
[9]

76 marks

SECTION B

QUESTION 7

You approach a bank for a home loan to the value of R850 000. This needs to be paid off monthly over a period of twenty years. The interest rate offered on the home loan is 9% per annum compounded monthly.

(a) What are the minimum monthly payments required to pay off the loan over the 20 year-period?

(4)

(b) If the monthly payment is changed to R9 000 per month, then what will the balance outstanding on your loan be at the end of 12 years?
(*This is directly after the payment is made at the end of 12 years.*)

(5)
[9]

QUESTION 8

(a) The first two terms of an arithmetic sequence are $\log_3(x-5)$ and $\log_3 x$. If the common difference of the sequence is 1 then what is the value of x?

(4)

(b) A converging geometric series is such that the second term is –24 and the sum of the third and fourth terms is –18. Determine the common ratio.

(5)

(c) You are the manager of a large sports stadium. A soccer tournament is held at your stadium where one match is played each day for 12 consecutive days.

The following rules and regulations have been given to you as the manager:

- At the first game a maximum of 25 000 spectators are allowed.
- For each subsequent game the number of spectators can increase by a constant amount so that the last game can have 58 000 spectators in the stadium.

Assume the maximum number of spectators attends each match.

(1) Determine the increase in the number of spectators between any two consecutive days of the tournament.

(3)

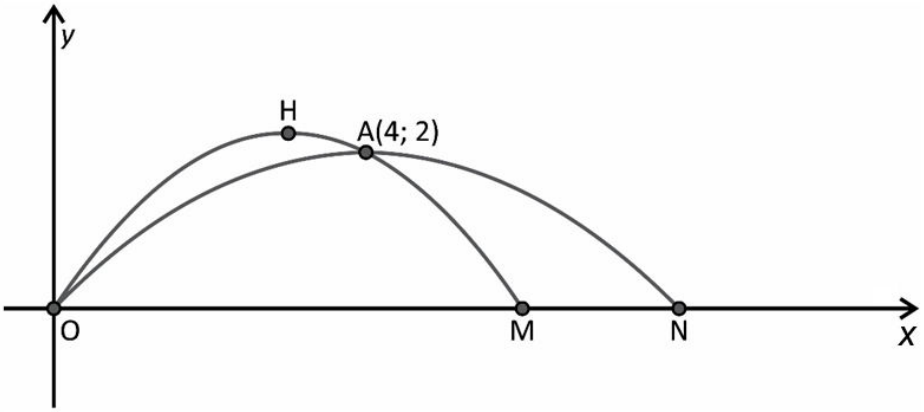
(2) If a spectator pays R25 for a ticket each day, then what is the total amount of income that can be generated from ticket sales, if all the rules and regulations above were followed?

(3)
[15]

QUESTION 9

(a) A sports scientist is looking at ways to throw a heavy object further. The diagram below looks at two throws and models them on the parabola.

- The first throw starts at O, it reaches its maximum height at H and lands at M.
- The second throw starts at O, reaches its maximum height at point A and then lands at N.



If the second throw is ***two metres further*** than the first throw, then what are the coordinates of point H?

(7)

(b) A quadratic function, $f(x) = ax^2 + bx + c$, has the following properties:

- $f(6-p) = f(6+p)$ for all real values of p.
- $f\left(-\frac{b}{2a}\right) = 2$
- $b^2 < 4ac$

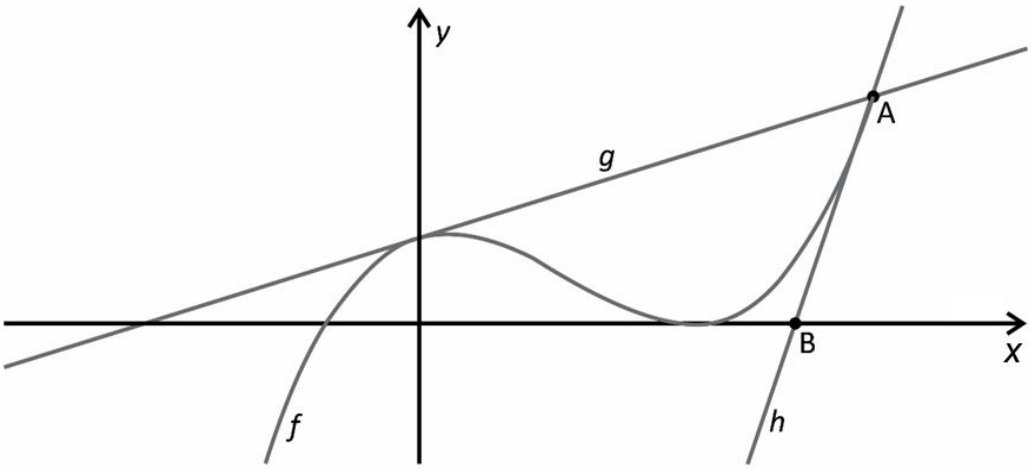
Draw a rough sketch graph of $f(x)$; labelling the turning point clearly.

(4)
[11]

QUESTION 10

(a) In the diagram below:

- $f(x) = x^3 - 5x^2 + 3x + 9$
- $g(x) = 3x + 9$
- f and g intersect at A



If $h(x)$ is a tangent at A, then calculate the coordinates of B, the x-intercept of h.

(9)

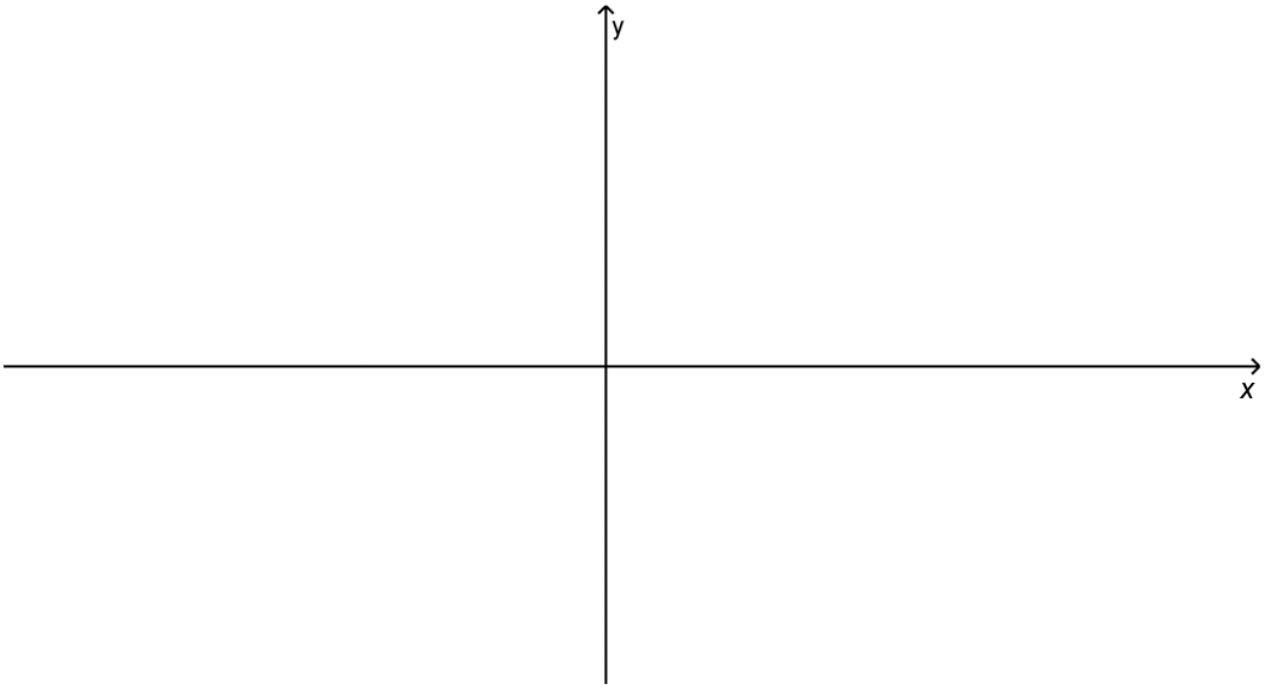
(b) The following information is given:

- $f'(x) = (x+1)^2 - 4$
- $f(x) = 0$ has one negative real solution.

(1) For what values of x is $f(x)$ increasing?

(4)

(2) Use the information above to sketch $f(x)$ on the set of axes below. Label the x value(s) of your turning point(s) and x value for the point of inflection.

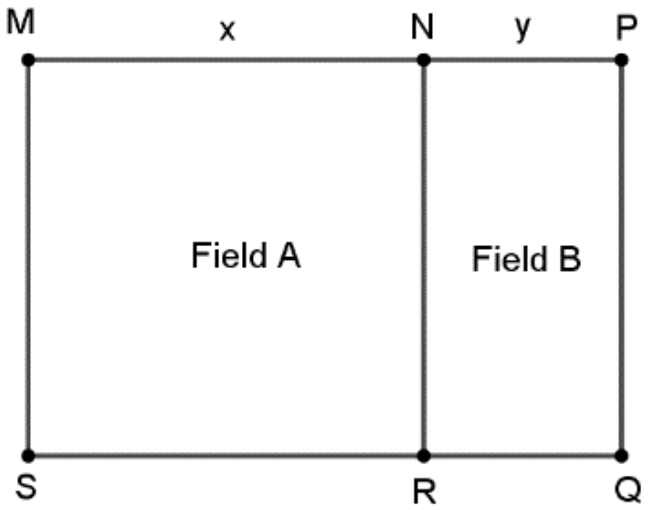


(5)
[18]

QUESTION 11

A farmer draws a rough sketch of the two adjacent fields he needs to fence.

- Field A is a ***square*** with a width of x.
- Field B is rectangular in shape with a width of y.
- Area of Field A + Area of Field B $= 1000$ units^2.
- NR is the only fence that is shared by both Field A and Field B.



(a) Find y in terms of x.

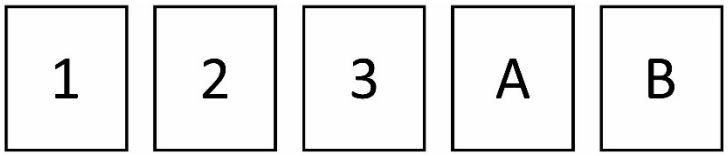
(3)

(b) Calculate the value for x so that the ***minimum*** amount of fencing is used.

(5)
[8]

QUESTION 12

(a) The five cards given below are used by players in a game to create five-digit codes.



A few examples of five-digit codes that can be created with the cards above are:

1A2B3; AB132; B2A13 …… (No repetition is allowed)

(1) How many unique five-digit codes can be created?

(1)

(2) How many unique five-digit codes starting with a number can be created?

(2)

(3) What is the ***probability*** that the two letters will ***never*** be next to each other in the code?

(4)

(b) You have three, six-sided die that you use to create a geometric series by randomly throwing the die. The one dice is red, the one dice is blue, and the third dice is green.

- The number on the red dice represents the value of the first term in the series.
- You then take the value on the blue dice and divide it by the value on the green dice to generate the common ratio.

What is the probability that if you sum the series to infinity it will converge on the number 10?

(6)
[13]

74 marks

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

ADDITIONAL SPACE

ADDITIONAL SPACE