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TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION  
NOVEMBER 2021

**MATHEMATICS: PAPER II**

EXAMINATION NUMBER

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Time: 3 hours

150 marks

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 28 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Three blank pages (pages 26 to 28) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Ensure that your calculator is in **DEGREE** mode.
8. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
9. It is in your own interest to write legibly and to present your work neatly.
10. Round off to **ONE DECIMAL PLACE** unless otherwise stated.

**FOR OFFICE USE ONLY: MARKER TO ENTER MARKS**

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	TOTAL
7	7	17	13	8	13	8	11	11	11	11	6	20	7	/150

**SECTION A**

**QUESTION 1**

The table illustrates the time spent ( $x$ ) by shoppers at a clothing store and the amount of money ( $y$ ) they spent on a visit to the clothing store.

<b>Time spent (<math>x</math>) in minutes</b>	<b>27</b>	<b>41</b>	<b>45</b>	<b>51</b>	<b>60</b>	<b>53</b>	<b>35</b>
<b>Money spent (<math>y</math>) in Rands</b>	<b>400</b>	<b>630</b>	<b>700</b>	<b>1 040</b>	<b>1 200</b>	<b>870</b>	<b>745</b>

- (a) Use your calculator to determine the equation for the line of best fit in the form  $y = A + Bx$ . **Give your answer correct to 3 decimal places.**

(3)

- (b) Predict how much an individual would spend if they were in the clothing store for 90 minutes. **Give your answer correct to 2 decimal places.**

(1)

- (c) Is the prediction in Question (b) a reliable one? Explain.

(1)

- (d) Calculate the correlation coefficient for the data. **Give your answer correct to 3 decimal places.**

(1)

- (e) What does the correlation coefficient suggest about the relationship between the time spent in a clothing store and the money spent by the individual?

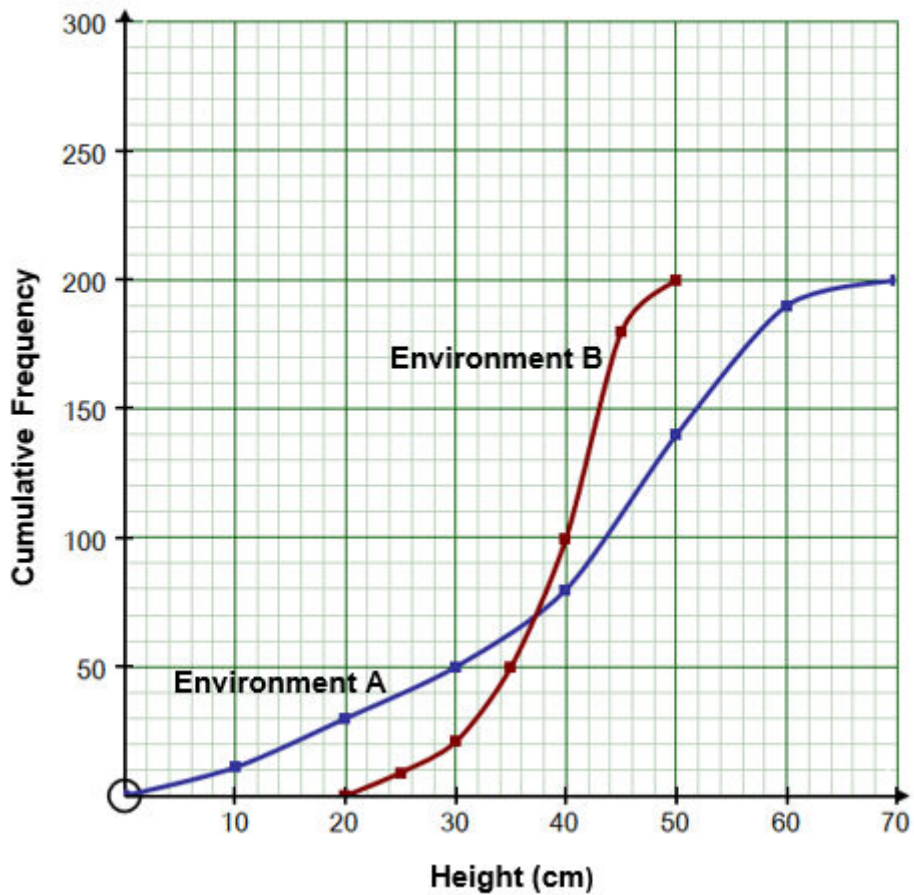
(1)  
[7]

**QUESTION 2**

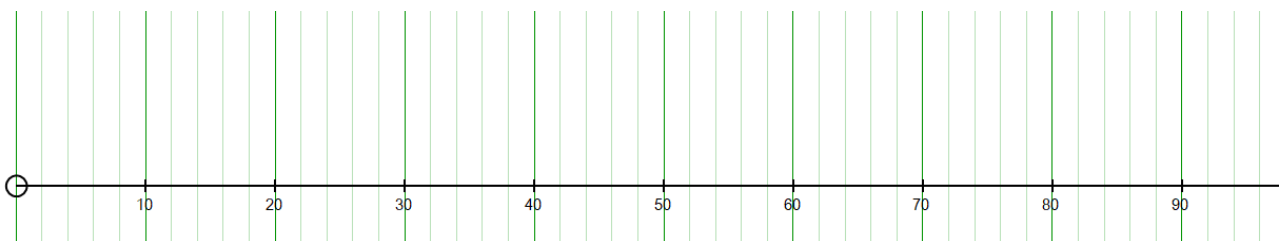
The diagram represents the ogive curves for the heights of two groups of plants. 200 plants are placed in Environment A and another 200 plants are placed in Environment B.

The minimum height of plants placed in Environment A is 2 cm and the maximum height is found to be 68 cm.

**Cumulative Frequency Curve (Ogive) for the heights of plants**



- (a) Draw a box and whisker plot that represents the height of plants in Environment A.



(5)

- (b) If the mean height of the plants in Environment B is given as 34 cm, describe the skewness of the data represented by the Environment B curve.

(1)

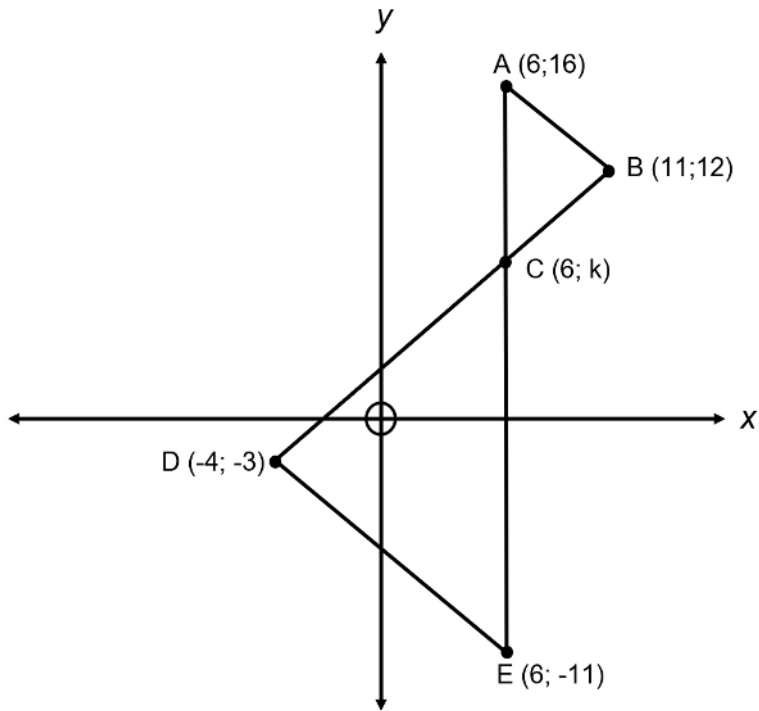
- (c) Under which environment were the heights of the plant more spread out? Explain.

(1)  
**[7]**

**QUESTION 3**

In the diagram:

- Points  $A(6;16)$ ,  $B(11;12)$ ,  $E(6;-11)$  and  $D(-4;-3)$  are given.
- $AE$  intersects  $DB$  at point  $C(6;k)$



(a) Calculate the length of  $AB$ .

(2)

(b) Show that  $AB \parallel DE$ .

(3)

(c) Determine the value of  $k$ .

(4)

(d) Determine the size of  $\hat{BAC}$ .

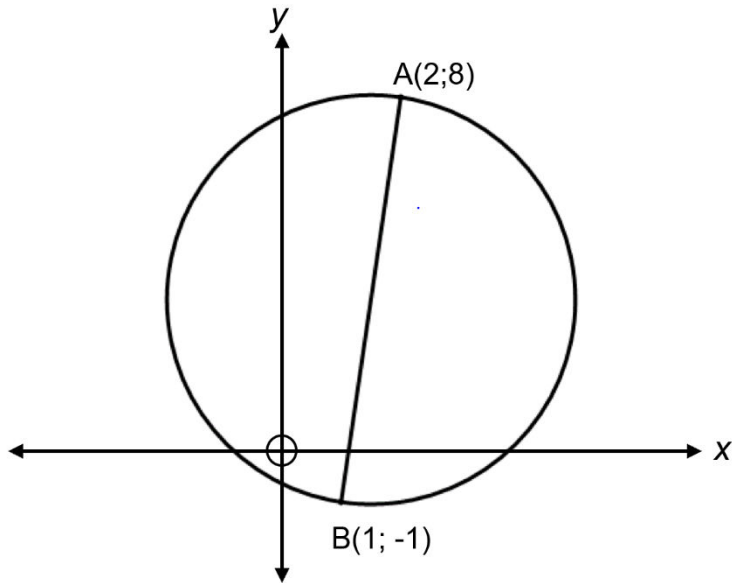
(3)

(e) Determine:  $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle EDC}$

(5)  
[17]

**QUESTION 4**

(a) In the diagram, AB is a diameter of the circle with A(2;8) and B(1;-1).



(1) Determine the length of AB.

(2)

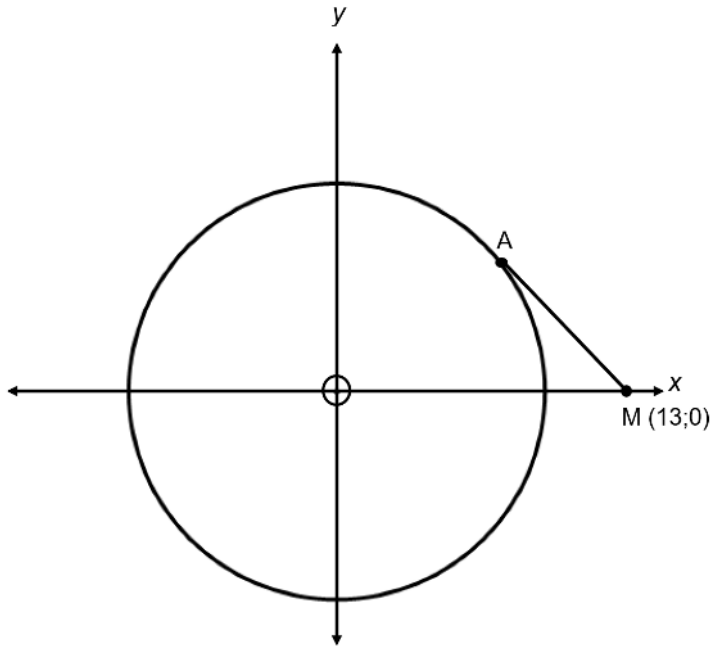
(2) Determine the equation of the circle.

(3)

(3) Determine the equation of the tangent to the circle at A.

(4)

- (b) In the diagram below, the circle  $x^2 + y^2 = 100$  is drawn. M is the point (13;0) and MA is a tangent to the circle.



Determine the length of the tangent AM. Leave your answer in surd form.

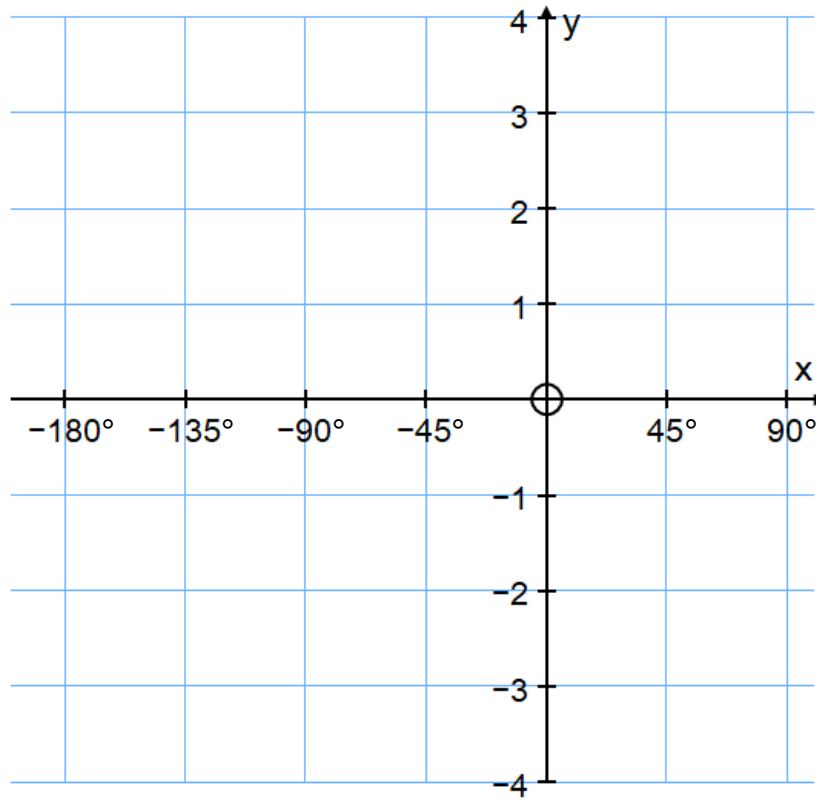
(4)  
[13]



**QUESTION 5**

Given:  $f(x) = \cos 2x$  and  $g(x) = \tan x + 2$  for  $x \in [-180^\circ; 90^\circ]$

- (a) Sketch the graphs of  $f$  and  $g$  on the same set of axes below. Label all intercepts with the axes, asymptotes, turning points and end points.



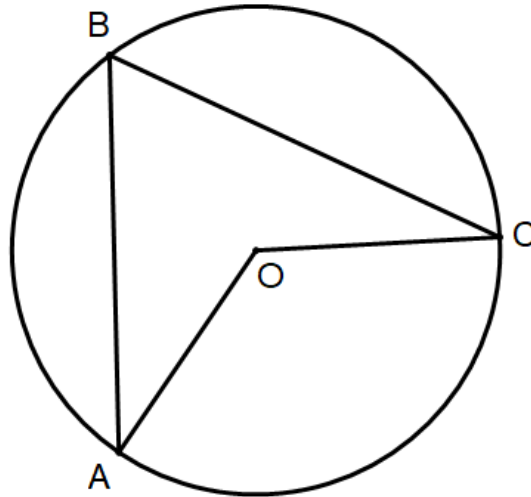
(6)

- (b) Use your graph to determine the approximate value(s) of  $x \in [-180^\circ; 90^\circ]$  for which  $\cos 2x - 2 \leq \tan x$ .

(2)  
[8]

**QUESTION 6**

- (a) Use the diagram below to prove the theorem that states that the **angle subtended by a chord at the centre of a circle is twice the size of the angle that it subtends at the circle.**



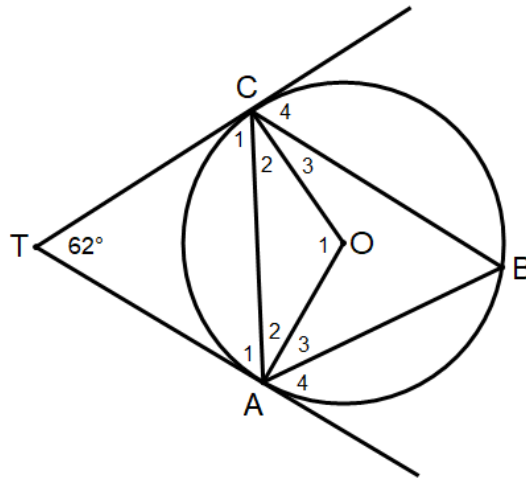
Required to prove:  $\hat{AOC} = 2 \times \hat{ABC}$

Construction:

Proof:

(6)

- (b) In the diagram:
- A, B and C lie on the circle with centre O.
  - TC and TA are tangents to the circle with  $\hat{CTA} = 62^\circ$ .



**State all relevant reasons with your statements.**

(1) Determine:  $\hat{A}_1$

(3)

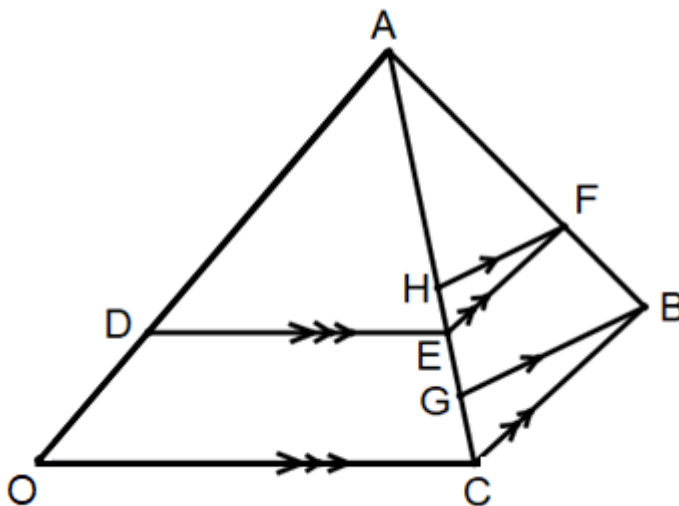
(2) Determine:  $\hat{O}_1$

(4)  
[13]

**QUESTION 7**

In the diagram,  $\triangle AOC$  and  $\triangle ABC$  are given.

- D lies on OA and E on AC with  $DE \parallel OC$ .
- F lies on AB with  $EF \parallel CB$ .
- H and G lie on AC with  $HF \parallel GB$ .
- $AO = 7$  units
- $AD = 4$  units

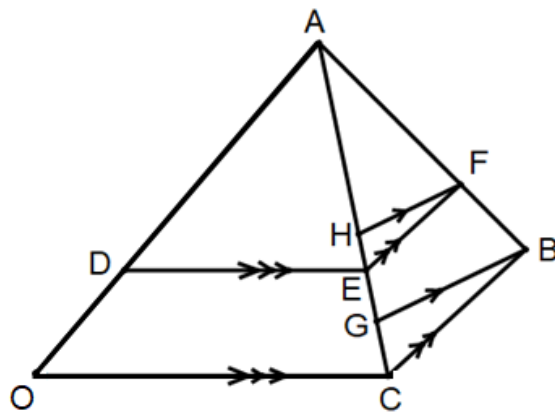


(a) Determine the ratio of  $AF : FB$ , giving reasons.

(3)

(b) Determine the ratio of  $GB : HF$ , giving reasons.

(3)



- (c) If it is further given that G is the midpoint of EC, determine the ratio AE : EG, giving reasons.

(2)  
[8]

**73 marks**

**SECTION B****QUESTION 8**

- (a) Solve for  $x$ , correct to one decimal place, in the given interval:

$$\sin 3x = -\frac{3}{4} \text{ for } x \in [-90^\circ; 0^\circ]$$

(4)

- (b) Determine the general solution for:

$$\tan x = \sin 2x \text{ if } \sin x \neq 0$$

(7)  
[11]

**QUESTION 9**

- (a) If  $\sin \hat{C} = \frac{12}{13}$  where  $0^\circ < C < 90^\circ$  and  $\tan \hat{D} = -\frac{4}{3}$  where  $270^\circ < D < 360^\circ$ , determine  $\sin(C - D)$  **without the use of a calculator.**

(6)

- (b) Given:  $\cos 58^\circ = k$ , express the following in terms of  $k$  **without the use of a calculator.**

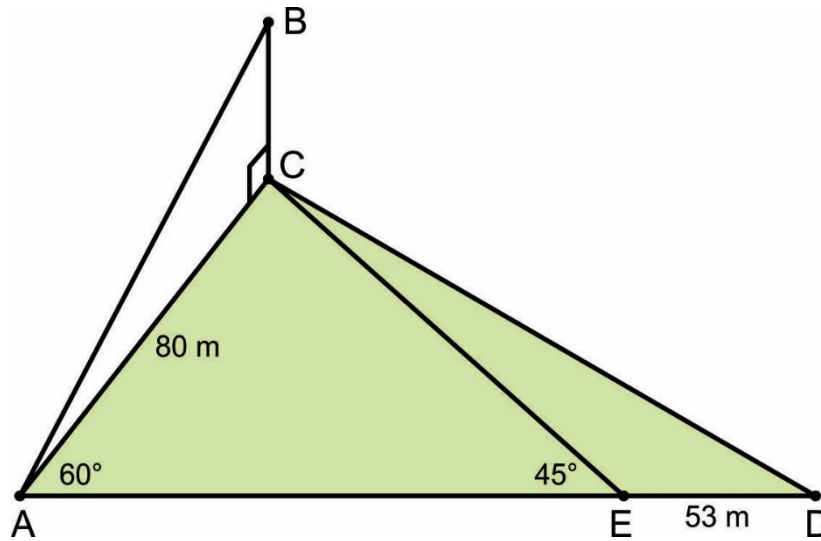
$$\cos 150^\circ \cdot \cos(-28^\circ) + \cos 60^\circ \cdot \cos 62^\circ$$

(5)  
[11]

**QUESTION 10**

In the diagram below:

- ACD is a triangular field on a horizontal ground.
- E is a point on AD.
- BC is a vertical building.
- AC = 80 m and ED = 53 m
- $\hat{CAE} = 60^\circ$  and  $\hat{CEA} = 45^\circ$



(a) Calculate the length of CD.



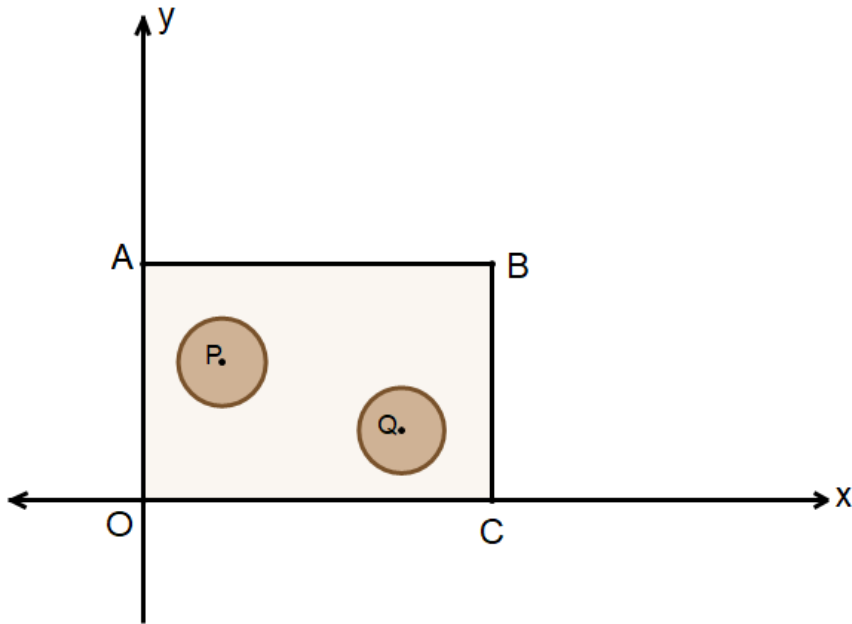
- (b) If the angle of elevation of B from A is  $37^\circ$ , determine the angle of elevation of the midpoint of BC from D.

(5)  
**[11]**

**QUESTION 11**

In the diagram below:

- $P(x ; y)$  and  $Q(9 ; 3)$  are the centres of two holes drilled into a rectangular piece of wood with base ABCO.
- The radii of the holes are equal.
- The equation of the circle with centre P is  $x^2 + y^2 - 6x - 12y + 41 = 0$ .
- The equations of AB and BC are  $y = 10$  and  $x = 14$  respectively.



(a) Determine the centre and the radius of the hole with centre P.

(4)

(b) Determine the shortest distance between the circles. Leave your answer in surd form.

(3)

(c) Calculate the volume of the block with base ABCO and height 20 units.

(4)  
[11]

**QUESTION 12**

The following set of 10 numbers is given where  $a > b$ .

$$\{a; a; a; a; a; b; b; b; b; b\}$$

- (a) Determine the mean in terms of  $a$  and  $b$ .

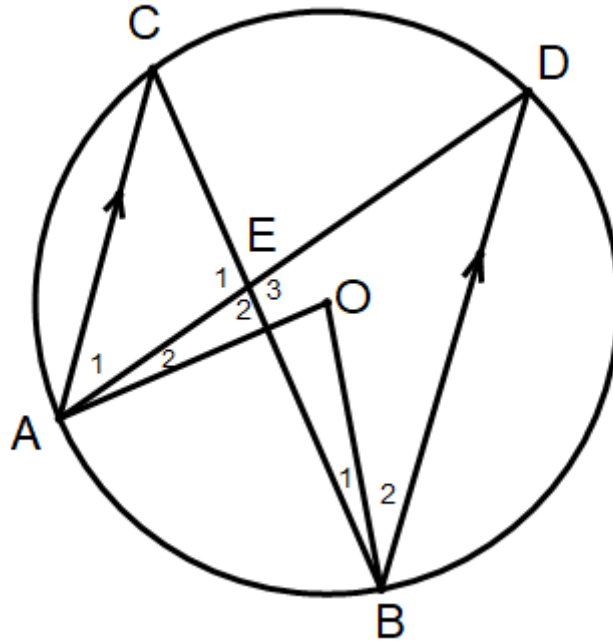
(1)

- (b) Determine the standard deviation in terms of  $a$  and  $b$ .

(5)  
[6]

**QUESTION 13**

- (a) In the diagram:
- O is the centre of the circle.
  - Chords AD and CB intersect at E.
  - $AC \parallel BD$

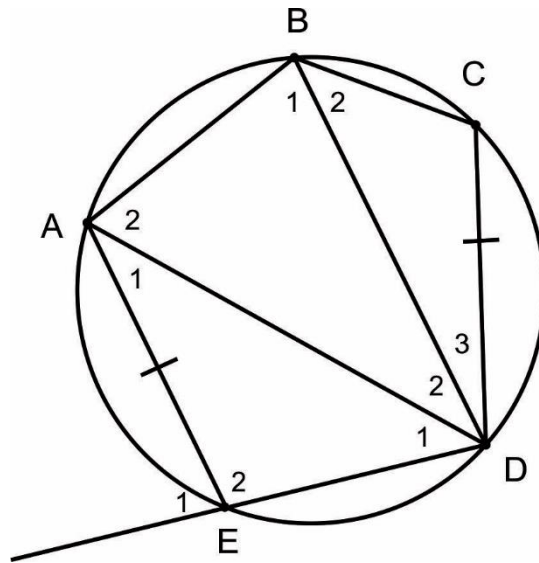


Prove that AEOB is a cyclic quadrilateral.

(7)

(b) In the diagram below

- A, B, C, D and E lie on the circle.
- $AE = CD$

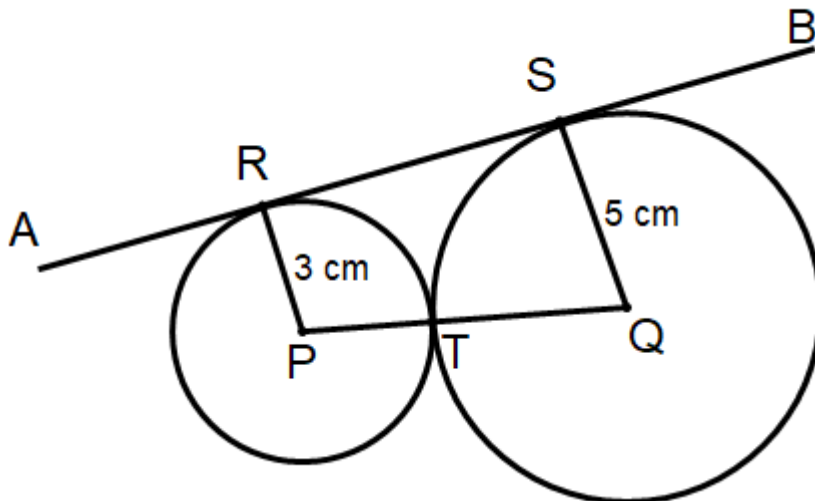


Prove that  $\hat{B}_1 - \hat{B}_2 = \hat{A}_1$

(6)

(c) In the diagram:

- P and Q are the centres of two circles.
- Circle P has a radius of 3 cm and circle Q has a radius of 5 cm.
- The two circles touch each other at T.
- AB is a common tangent to the circles at R and S.



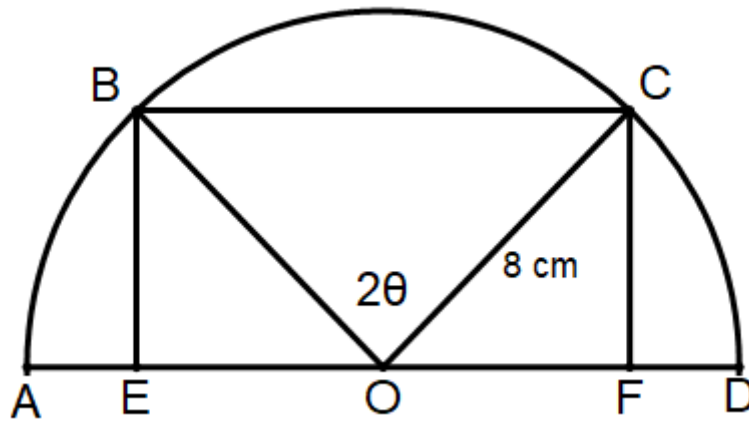
Determine the length of RS.

(7)  
[20]

**QUESTION 14**

In the diagram,

- Rectangle EBCF is inscribed in a semicircle with centre O.
- The radius of the semicircle is 8 cm.
- $\hat{BOC} = 2\theta$



- (a) If the perimeter of the rectangle EBCF is represented by  $P$ , show that  $P = 16\cos\theta + 32\sin\theta$ .

(4)



- (b) If it is further given that  $P = 16\sqrt{5} \sin(\theta + \alpha)$  and  $\alpha$  is an acute angle, determine the value of  $\alpha$ .

(3)  
[7]

**77 marks**

**Total: 150 marks**

**ADDITIONAL SPACE (ALL questions)**

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**



