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TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2022

MATHEMATICS: PAPER II

EXAMINATION NUMBER

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. Round off to **ONE DECIMAL PLACE** unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. ONE blank page (page 28) is included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	TOTAL
10	17	12	6	10	10	8	10	22	7	11	9	10	8	/150

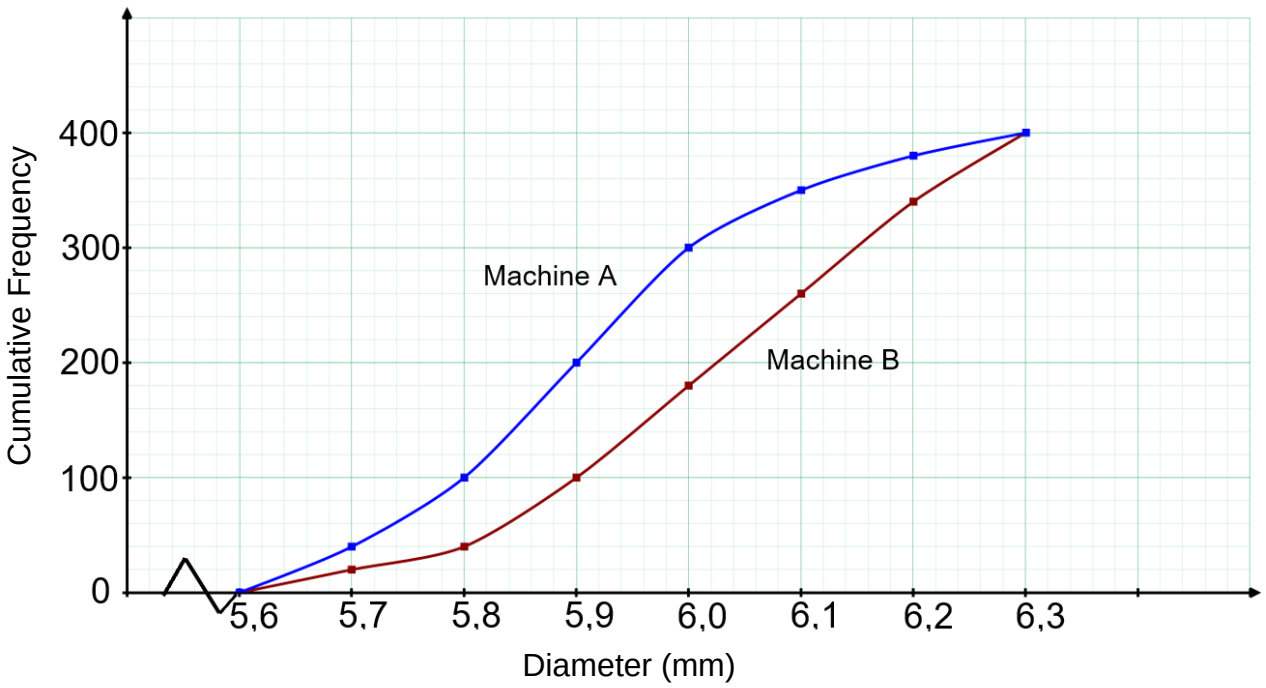
SECTION A

QUESTION 1

(a) A jewellery manufacturer measured the diameters of 400 gold spheres produced by machine A and another 400 produced by machine B.

The results are represented in the cumulative frequency curves below.

Cumulative Frequency (ogive) Curves for the diameters of gold spheres produced by Machine A and Machine B



(1) Determine the interquartile range for the diameters of gold spheres produced by machine A.

(3)

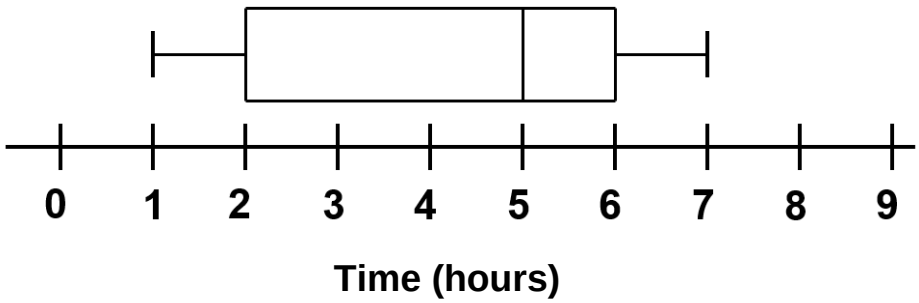
(2) Determine the 50^{th} percentile for the diameters of gold spheres produced by machine A.

(1)

(3) If the gold spheres with diameters less than or equal to 5,9 mm are considered defective, determine the percentage of defective gold spheres produced by machine B.

(1)

(b) In a school, learners were surveyed about the time they spent on the internet over a weekend. The results were displayed by a box and whisker plot, as shown below.



(1) Is the data positively or negatively skewed?

(1)

(2) Consider the statement given:

The percentage of learners that spent from 2 to 5 hours on the internet is almost three times as much as the percentage of learners that spent from 6 to 7 hours on the internet.

Is the statement true or false?

(1)

(3) One of the learners was not included in the data represented in the box and whisker plot above. If the learner spent 10 hours on the internet over the weekend, then determine whether the learner is an outlier or not.

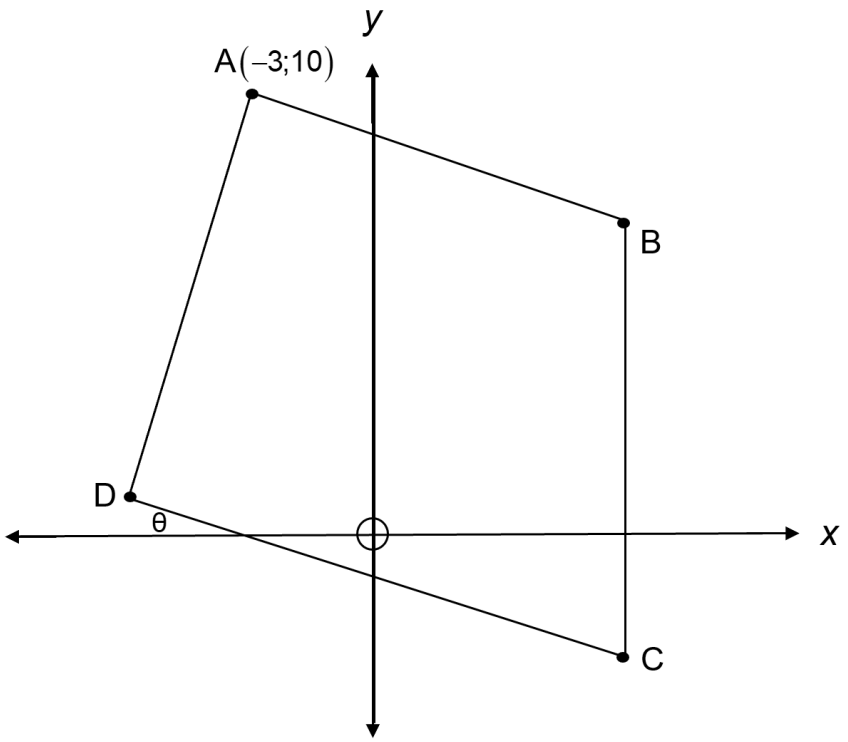
Use the formula $Q_3 + 1{,}5 \times \text{IQR}$.

(3)
[10]

QUESTION 2

In the diagram:

- Point $A(-3;10)$ and B in the first quadrant, are points on quadrilateral ABCD.
- Equation of line DC is given as: $y = -\frac{1}{3}x - 1$.
- DC forms an angle θ with the *x*-axis as indicated.



(a) Determine the size of θ.

(2)

(b) Determine the equation of the line AB if $AB \parallel DC$.

(3)

(c) Determine the equation of the line AD if $AD \perp DC$.

(3)

(d) (1) Determine the coordinates of D and hence calculate the length of AD.

(5)

(2) If it is further given that the equation of the line BC is $x = 6$, show that ΔABD is isosceles.

(4)
[17]

QUESTION 3

(a) The test scores of 5 students are given in ascending order as: $\{25; x+3; x+6; x+9; x+13\}$. The median is 41.

(1) Determine the value of x.

(2)

(2) Determine the standard deviation of the scores using your answer to (a)(1).

(2)

(3) How many scores will lie within one standard deviation from the mean? Show your working.

(3)

(b) The minimum overnight temperatures ($T\,°C$) and the number of service calls (S) made to a company that supplies gas heaters was recorded for a period of 8 days.

The equation of the least squares regression line for the data is given as: $S = -1{,}8T + 22{,}7$

(1) State whether the data represents a **positive** or **negative** correlation.

(1)

(2) Use the regression line to predict the number of service calls made, for an overnight temperature of 10 °C.

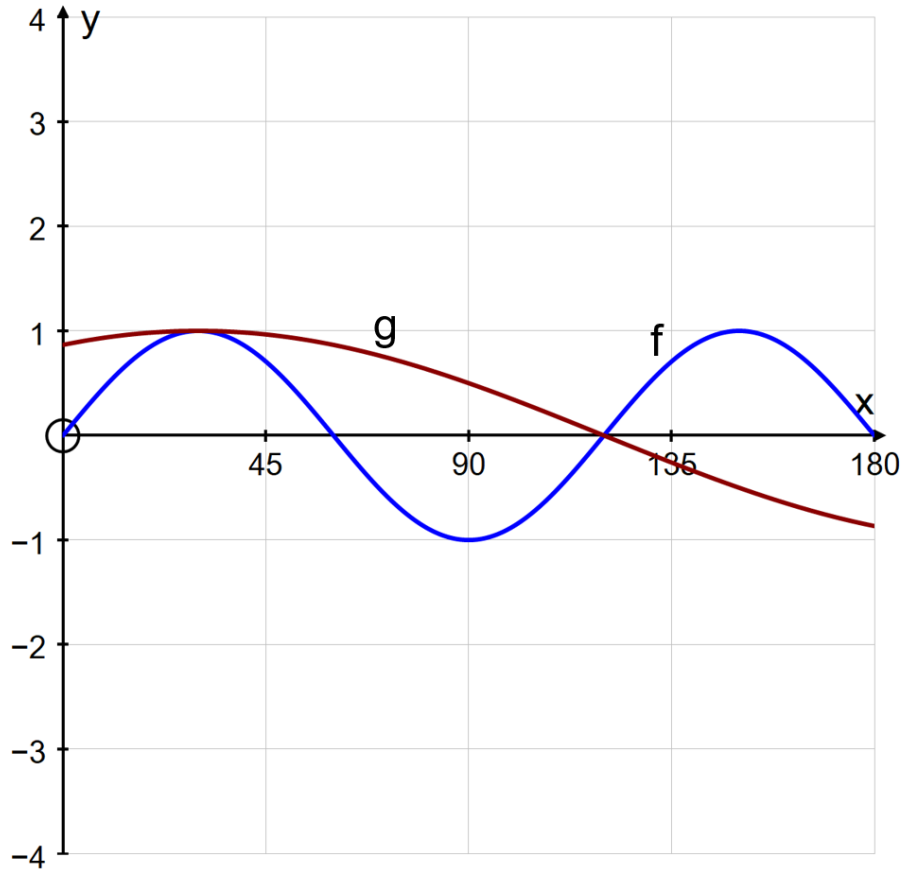
(2)

(3) The correlation coefficient for the 8-day period is $-0,95$. On the 9^{th} day, the number of service calls was 8 and the minimum overnight temperature was 3 °C. If the 9^{th} day data is included, what effect will it have on the correlation coefficient? Explain.

(2)
[12]

QUESTION 4

Given: $f(x) = \sin 3x$ and $g(x) = \cos(x - 30°)$ for $x \in [0°;180°]$



(a) Determine the period of the graph of f.

(1)

(b) Determine the range of the graph of g.

(2)

(c) Show on your graph (**using capital letters**), for $x \in [0°;180°]$ the solution(s) to:
$\sin 3x = \cos(x - 30°)$

(1)

(d) Use your graph to determine the solution(s) to:

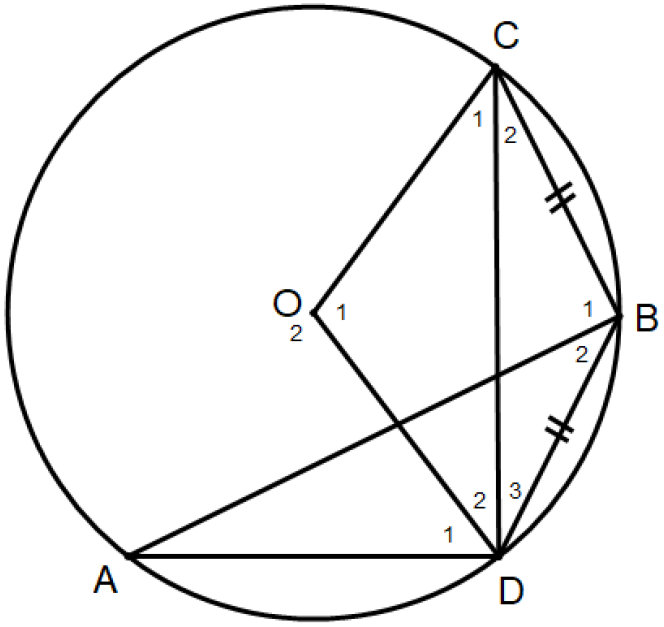
$\cos(x - 30°) \geq \sin 3x$ for $x \in [0°;180°]$

(2)
[6]

QUESTION 5

In the diagram:

- A, D, B and C lie on the circle with centre O.
- DB $=$ BC.



State all relevant reasons with your statements.

(a) If $\hat{A} = x$, state, with reasons, two other angles equal to x.

(3)

(b) (1) Given $\hat{O}_1 = 94°$, determine $\hat{D}_2$.

(2)

(2) Determine $\hat{B}_1 + \hat{B}_2$.

(3)

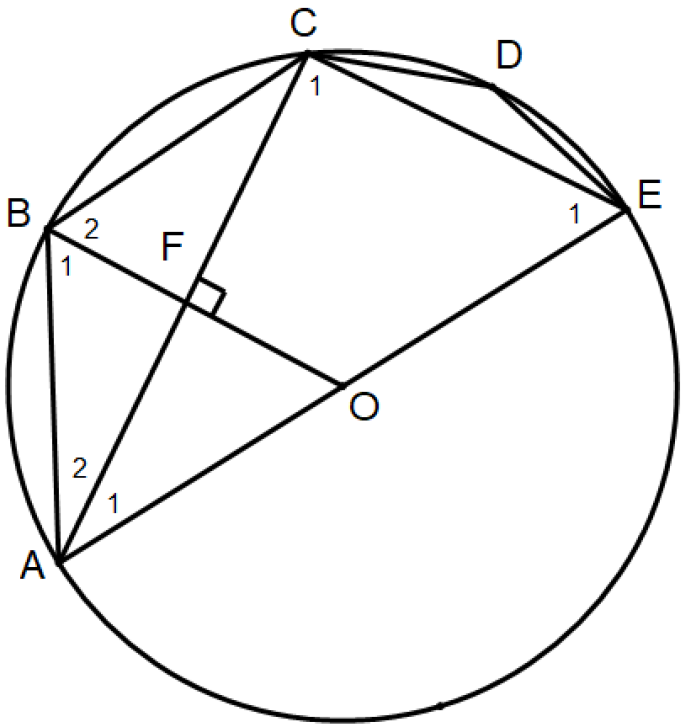
(3) Hence or otherwise determine x.

(2)
[10]

QUESTION 6

In the diagram:

- A, B, C, D and E lie on the circle with centre O.
- AC is perpendicular to OB and they intersect at F.
- AOE is a straight line.
- $\hat{A}_1 = 38°$.



State all relevant reasons with your statements.

(a) Determine $\hat{C}_1$.

(2)

(b) Determine $\hat{D}$.

(2)

(c) Determine $A\hat{B}C$.

(3)

(d) If AC = 8 cm and BC = 5 cm, determine the length of BF.

(3)
[10]

QUESTION 7

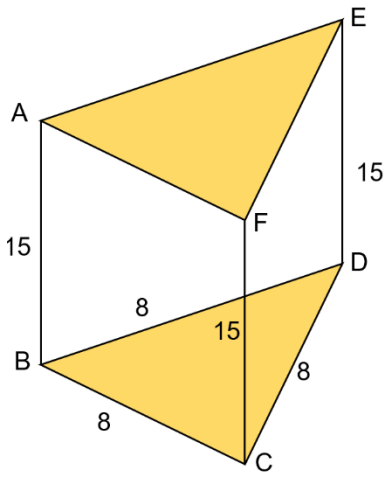
(a) Prove the area-rule stating that:

Area $\Delta PQR = \frac{1}{2}pq\sin\hat{R}$ for any acute angled triangle ΔPQR.

(4)

(b) In the diagram, a solid right triangular prism with sides 8 cm and height 15 cm is given.

Calculate the total surface area of the prism.



(4)
[8]

73 marks

SECTION B

QUESTION 8

(a) Solve for x, correct to one decimal place in the given interval.

$$1-2\sin^2 x = -\frac{1}{7} \quad \text{for} \quad x \in [-180°;90°]$$

(5)

(b) Simplify the following to its simplest form:

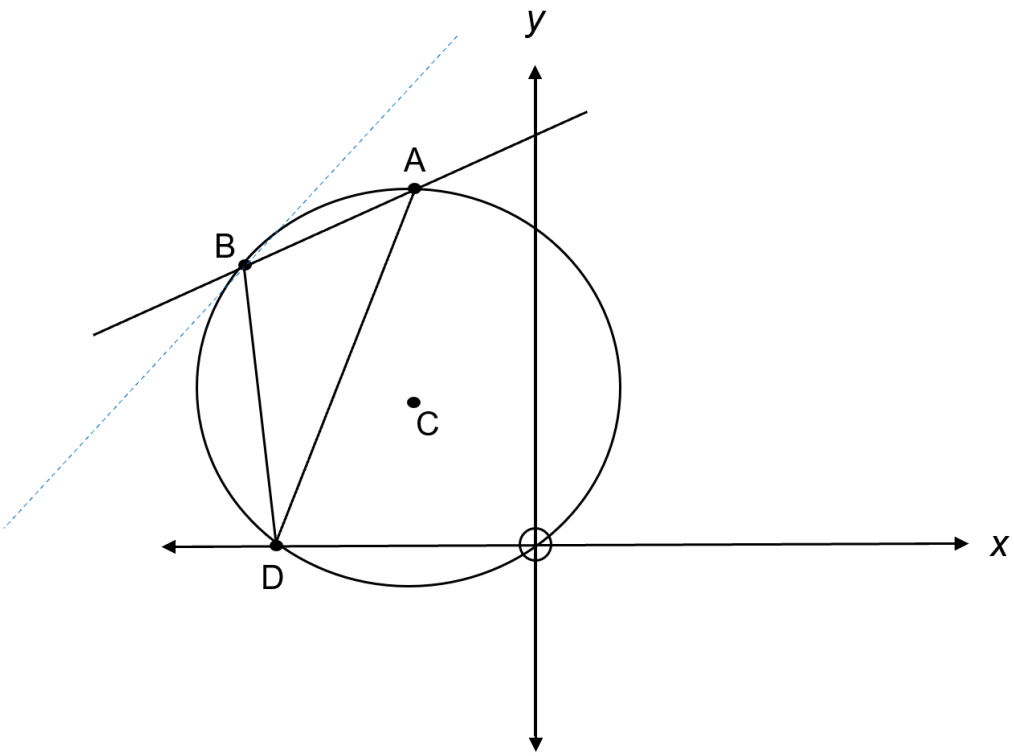
$$-\cos(\theta).\cos^3(90^\circ+\theta)-\tan(180^\circ-\theta).\cos(\theta).\cos^3(-\theta)$$

(5)
[10]

QUESTION 9

In the diagram below:

- A, B, D and the origin O lie on the circle with centre C.
- The equation of the circle is: $x^2 + y^2 + 6x - 8y = 0$.
- The equation of line AB is: $x - 2y + 21 = 0$.



(a) Determine the centre and the radius of the circle.

(4)

(b) Determine the co-ordinates of A and B.

(6)

(c) (1) Determine the midpoint of AD.

(4)

(2) Determine whether CB passes through the midpoint of line AD.

(4)

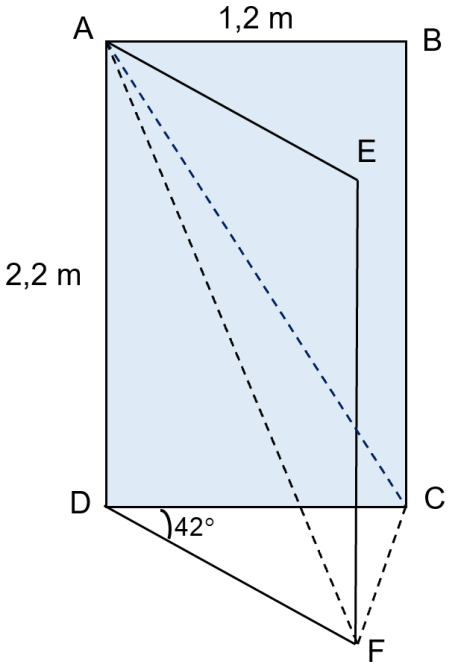
(d) A second circle with equation $(x-3)^2+(y+4)^2=25$ is drawn on the same set of axes. A student claims that the two circles touch each other in one point. Is the student correct? Show all working.

(4)
[22]

QUESTION 10

In the diagram below:

- ABCD represents a closed door 1,2 metres wide and 2,2 metres in length.
- When the door is open $42°$ along its hinge AD, its new position is represented by AEFD.



Determine the size of $F\hat{A}C$.

[7]

QUESTION 11

(a) Prove the following identity:

$$\frac{1+\sin 2x+\sin^2 x-\cos^2 x}{1+2\sin x\cos x+\cos 2x}=\tan x$$

(5)

(b) Determine the values of x for which the identity given in (a) is not valid.

(6)
[11]

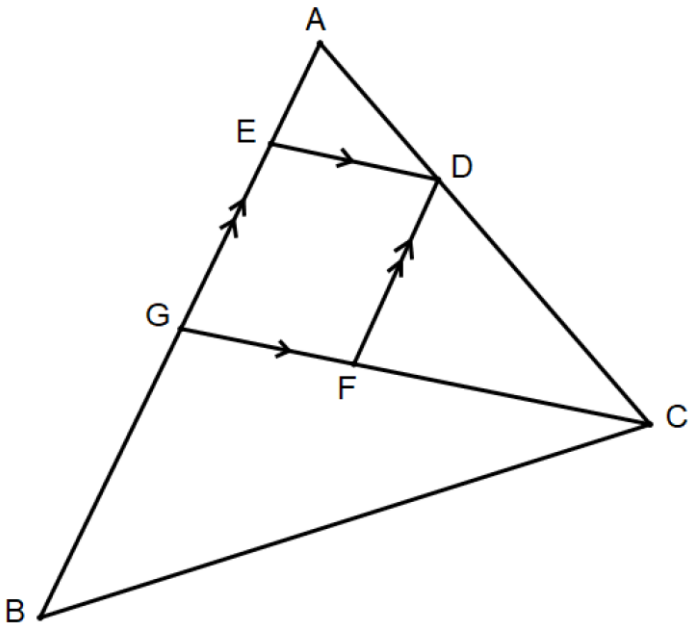
QUESTION 12

ΔABC is given with E and G on AB and D on AC with:

- $AG = GB$
- $ED \parallel GC$

F is on CG and D on AC with:

- $DF \parallel AB$



(a) Prove: $\frac{CF}{FG} = \frac{GE}{EA}$

(3)

(b) If $\frac{CF}{FG} = \frac{2}{1}$, determine the ratio of $GE : GA$.

(2)

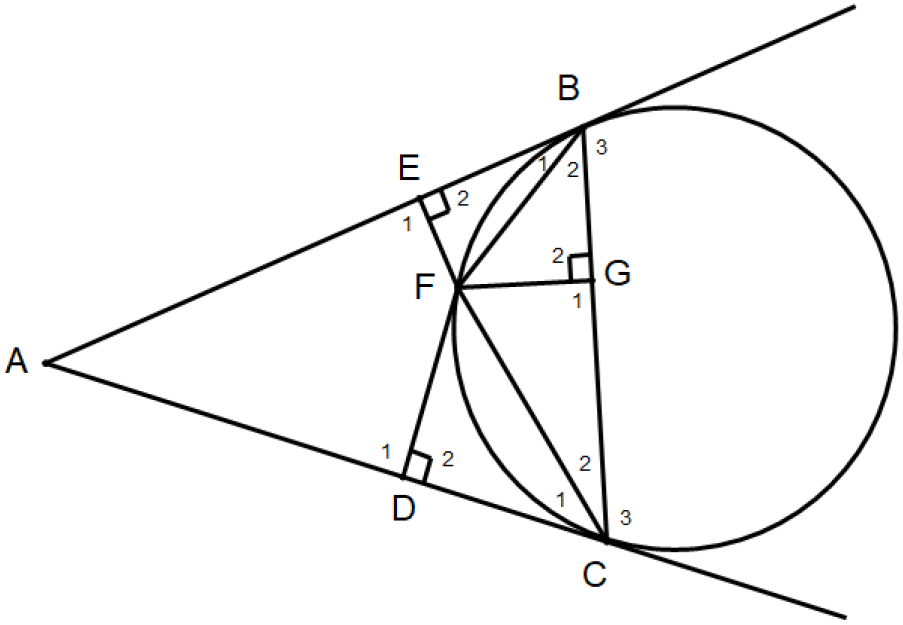
(c) Determine the numerical value of $DF : AB$.

(4)
[9]

QUESTION 13

In the diagram the circumscribed circle for ΔBCF is given.

- AB and AC are tangents to the circle at B and C respectively.
- FE, FG and FD are perpendiculars from F to AB, F to BC and F to AC.



(a) Prove: ΔFEB is similar to ΔFGC

(4)

(b) Prove: $FG^2 = FE \times FD$

(6)
[10]

QUESTION 14

(a) If $2\cos 2\alpha + \sin 2\alpha = R\cos(2\alpha - \beta)$ where $R > 0$ and β is an acute angle.

Determine R and β.

(6)

(b) Hence, or otherwise, determine the maximum value of $4\cos^2 \alpha + \sin 2\alpha$.

(2)
[8]

77 marks

Total: 150 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.